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Modeling of 2-DOF Robot Arm and Control

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Abstract

The mathematical modeling of two degrees of freedom robot arm (2-DOF) is developed and presented in this paper. The model is based on a set of nonlinear second-order ordinary differential equations and to simulate the dynamic accurately lagrangian and Euler-Lagrange equations were successfully derived and established. The control algorithm is expanded on the derived mathematical equations to control the robot arm in joint angle position and the coupling effect of the robot arm was decoupled so as to gain sufficient freedom to control each arm freely. Proportional-integral-derivate controllers (PIDs) was implemented in the model and the simulation model was developed with the aid of MATLAB and Simulink R2014b version 8.4 simulation tool to investigate the system performance in joint space. According to the results analysis, the robot arm was satisfactorily controlled to reach and stay within a desired joint angle position through implementation and simulation of PID controllers using MATLAB/Simulink. This model serves as simulation platform to test the performance of the robot arm with different joint angles position and to observe the responses prior to the implementation the model in the actual robot arm.

Keywords: Modeling, 2-DOF robot arm, PID controller, Lagrangian and Euler-Lagrange, MATLAB/Simulink

1. Introduction

In this technology-driven economy, the demand for the robot is increasing rapidly and its applications are widespread across all sectors. The study of robot arm control has gained a lot of interest in manufacturing industry, military, education, biomechanics, welding, automotive industry, pipeline monitoring, space exploration and online trading (Mohammed, 2015; O zkan, 2016; Rajeev Agrawal, Koushik Kabiraj, 2012; Salem, 2014; Virgala, 2014) due to the fact that it works in unpredictable, dangerous, and hostile circumstances which human cannot be reached. Recently, the robot arm is on increasing demand in health services to administer drugs to patients and rehabilitate the disabled and aged people; of which high accuracy and precision with zero-tolerance to error are of high significance for efficient utilization. (Paper, Wongphati, and Co, 2012; Virendra and Patidar, 2016).

A robot arm is a kind of mechanical device, programmable, multi-functional manipulator (Sanchez-Sanchez and Reyes-Cortes, 2010) designed with an intention to interact with the environment in a safe manner. It is a mechanical device in the sense that it has links and joint that provide stability and durability but are redundant from a kinematic perspective since the forces involve in the motion are not considered. The problems of high non-linearity in the coupling reaction forces between joints, as result of coupling effect and inertia loading(Craig, 2005; Munro, 2004; Virgala, 2014) are not well captured from the kinematics perspective. However, in-depth understanding of dynamic modeling is essential to address the controlling problem associated with the robot arm.

Modeling, simulation and control of robot arm had received tremendous attention in the field of mechatronics over the past few decades and the quest for new development of robot arm control still continues. In literature (Mohammed, 2015), kinematics model of a 4-DOF robot arm is addressed using both Denavit-Hartenberg (DH) method and product of exponential formula; and the result under study has shown that both approaches resulted in an identical solution. In the study (Gea and Kirchner, 2008),the impedance control is implemented to control the interaction forces of a simulated 2 link planar arm; a mathematical model of a robot is modelled, linearized and decoupled in order to establish a model-based controller. Simmechanic is used as a simulation tool to model the mechanics of the robot which permit the possibility to vary model-based control algorithms. The fundamental and concepts of 5 DOF of educational robot arm study in (Mohammed Abu Qassem, Abuhadrous, & Elaydi, 2010) to promote the teaching of the robot in higher institution of learning. To achieve this, a detailed kinematic analysis of an ALSB robot arm was investigated and a graphical user interface (GUI) platform was developed with Matlab programming language which also includes on-line motional simulator of the robot arm to fascinate and encourage experimental aspect of robot manipulator motion in real time among undergraduates and graduates.

The research work in (Virgala, 2014) centred on analysing, modelling and simulation of humanoid robot hand from the perspective of biology focusing on bones and joints. A new method for the inverse kinematic model is introduced using Matlab functions and dynamic model of humanoid hand is established using model-based design with aid of Matlab/Simmechanics. The conclusion of their work is that they established a model in Matlab which can be used to control finger motion. The author in (Lafmejani and Zarabadipour, 2014) modeled, simulated and controlled 3-DOF articulated robot manipulator by extracting the kinematic and dynamic equations using Lagrange method and compared the derived analytical model with a simulated model using Simmechanics toolbox. The model is further linearized with feedback and a PID controller is implemented to track a reference trajectory. It was concluded in the research work that robot manipulator is difficult to control as result of complexity and nonlinearity associated with the dynamic model.

Mahil and Al-durra, (2016), presented a linearized mathematical model and control of 2-DOF robotic manipulator and derived a mathematical model based on kinematic and dynamic equations using the combination of Denavit Hartenberg and Lagrangian methods. In his work, two different control strategies were implemented to compare the performance of the robot manipulator.

According to Salem, (2014), a robot arm model and control issues based on Simulink for educational purpose is presented. It established a comprehensive transfer function for both the motor and the robot arm which provide an insight into the dynamic behaviour of the robot arm. It later proposed a model for research and education purposes; which is used to select and analyze the performance of the system both in open and closed loop systems. (Razali, Ishak, Ismail, Sulaiman, Ismail and Al, (2010), employed 2-DOF robot arm for agricultural purposes such as planting and harvesting and computer simulation based on visual basic is developed which enable the users to control the way the robot moves and grab selected target according to real line situation. Many authors (Mailah, Zain, Jahanabadi, and A, 2009; Manjaree, 2017; Salem, 2014) developed a model for the robot arm and controlled the dynamic response of the robot arm using Simmechanics as a software tool. However, detail essential functions of each block that describe the mathematical model of the dynamic equations are not well captured with Simmechanics.

The accurate control of motion is a fundamental concern in the robot arm, where placing an object in the specific desired location with the exact possible amount of force and torque at the correct definite time is essential for efficient system operation. In other words, control of the robot arm attempts to shape the dynamic of the arm while achieving the constraints foisted by the kinematics of the arm and this has been a key research area to increase robot performance and to introduce new functionalities. In general, the control problem involves finding suitable mathematical models that describe the dynamic behaviour of the physical robot arm for designing the controller and identifying corresponding control strategies to realize the expected system response and performance. New strategies for controlling the robot arm has been more recently introduced such as PID (David & Robles, 2012; Guler & Ozguler, 2012; Lafmejani & Zarabadipour, 2014; Rajeev Agrawal, Koushik Kabiraj, 2012), Fuzzy logic and Fuzzy pattern comparison technique (Bonkovic, Stipanicev, & Stula, 1999), Impedance control (Gea & Kirchner, 2008; Jezierski, Gmerek, Jezierski, & Gmerek, 2013), LQR Hybrid control (Humberto, Rojas, Serrezuela, Adrian, Lopez, Lorena & Perdomo, 2016), GA Based adaptive control (Vijay, 2014), neuro-fuzzy controller (Branch, 2012) and Neural networks (Pajaziti & Cana, 2014). The objective of this research is to establish a mathematical model which represents the dynamic behaviour of the robot and effectively control the joint angle of the robot arm within a specified trajectory.

2. Methodology

The dynamics of 2-DOF robot arm was modelled using a set of nonlinear, second-order, ordinary differential equations and to simulate the dynamics accurately the Lagrangian and Lagrange-Euler was adopted. The Euler's formulation is chosen for its simplicity, robustness (Amin, Rahim, & Low, 2014) energy based property (David & Robles, 2012),easy determination and exploitation of dynamic structural property and minimal computational error as compared to Newton-Euler approach (Murray, 1994) to solve the derived mathematical model. The formulation of the mathematical model is considered crucial in the research because the control strategy is investigated based on these derived dynamics equations, hence the model must be accurately predicted to represent the dynamic behaviour of the robot arm. The control algorithm is expanded on the derived mathematical model to control the movement of the robot arm within the specified trajectory or workspace, hence, we further design a PID controller and tuned the PID based on trial and error method to obtain suitable controller parameters for proper controlling of the robot arm within the specified trajectory. Simulation studies based on MATLAB and Simulink are performed on the robot arm taken into the consideration the obtained PID controller parameters and the obtained parameters are used to validate the mathematical model in the joint space. The evaluation of the results obtained is presented and discussed extensively concerning achievement as well as providing recommendations for further work.

2.1 Mathematical Model of 2-DOF Robot Arm

The dynamics of a robot arm is explicitly derived based on the Lagrange-Euler formulation to elucidate the problems involved in dynamic modelling. Figure 1 shows the schematic diagram of two degree of freedom (DOF) of the robot arm with the robot arm link1 and link 2, joint displacement are θ_1 and θ_2,link lengths are $l_1$and l_2, m_1,m_2 represent the masses of each link and τ_1 and τ_2 are torque for the link 1and 2 respectively. In the model, the following assumptions are made:

i. The actuators dynamics (motor and gear boxes) is not taken into account.
ii. The effect of friction forces is assumed to be negligible
iii. The mass of each link is assumed to be concentrated at the end of each link.

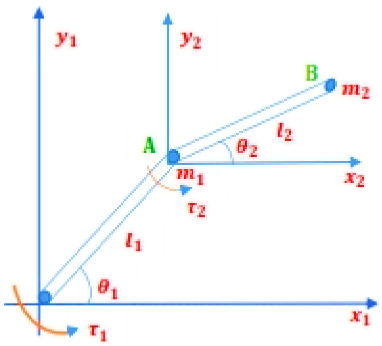


Figure 1: 2-DOF robot arm

First, the kinetic and the potential energies of the system are calculated, the kinetic energy of the manipulator as function of joint position and velocity is expressed as:

$$K(\theta,\dot{\theta}) = \frac{1}{2}\,\dot{\theta}^T M(\theta)\theta^{\cdot} = \frac{m_i v^2}{2} \qquad (1)$$

where, $M(\theta)$ is the nxn manipulator mass matrix and the subscript i denote 1and 2.
Hence, the total kinetic energy of the robot arm is the sum of the kinetic energies (K_1 and K_2) of the individual link.

$$K(\theta,\dot{\theta}) = \sum_{i}^{n} K_i\,(\theta,\dot{\theta}) \qquad (2)$$

$$K(\theta,\dot{\theta}) = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \qquad (3)$$

To calculate K_1 and K_2, are differentiated the position equations for m_1 at A as well as m_2 at B are written and subsequently, we differentiate the respective positions using inner product to obtain their corresponding velocity.

$$x_1 = l_1 \sin\theta_1 \qquad (4)$$

$$y_1 = l_1 \cos\theta_1 \qquad (5)$$

$$x_2 = l_1 \sin\theta_1 \; + \; l_2 \sin(\theta_1 + \theta_2) \qquad (6)$$

$$y_2 = -l_1 \cos\theta_1 - l_2 \cos(\theta_1 + \theta_2) \qquad (7)$$

Considering velocity, it is defined as,

$$v = \frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \qquad (8)$$

$$v^2 = \|v\|^2 = v^T v \quad (9)$$

$$v_1^2 = \begin{bmatrix} l_1 \cos\theta_1\dot{\theta}_1 & -l_1 \sin\theta_1\dot{\theta}_1 \end{bmatrix} \begin{bmatrix} l_1 \cos\theta_1\dot{\theta}_1 \\ -l_1 \sin\theta_1\dot{\theta}_1 \end{bmatrix} \quad (10)$$

$$v_1^2 = l_1^2(\cos\theta_1^2 + \sin\theta_1^2)\dot{\theta}_1 = l_1^2\,\dot{\theta}_1^2 \quad (11)$$

Similarly, v_2^2 is computed in the same view

$$v_2^2 = l_1^2\,\cos\theta_1^2\,\dot{\theta}_1^2 + 2l_1l_2\cos\theta_1\cos(\theta_1+\theta_2)\left(\dot{\theta}_1^2 + \dot{\theta}_1\,\theta_2\right) + l_2^2\,\cos^2(\theta_1+\theta_2)\left(\dot{\theta}_1+\dot{\theta}_2\right)^2$$
$$+ l_1^2\,\sin\theta_1^2\,\dot{\theta}_1^2 + 2l_1l_2\sin\theta_1\sin(\theta_1+\theta_2)\left(\dot{\theta}_1^2 + \dot{\theta}_1\,\theta_2\right)$$
$$+ l_2^2\,\sin^2(\theta_1+\theta_2)\left(\dot{\theta}_1+\dot{\theta}_2\right)^2 \quad (12)$$

To reduce the complexity in the derivation, we denote the trigonometry as:

$$sin\theta_1 = s_1, \sin(\theta_1+\theta_2) = s_{12}, \cos\theta_1 = c_1, \cos(\theta_1+\theta_2) = c_{12}$$

$$v_2^2 = l_1^2\,\dot{\theta}_1^2 + l_2^2\left(\dot{\theta}_1 + \dot{\theta}_2 + 2\dot{\theta}_1\dot{\theta}_2\right) + 2l_1l_2c_2\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right) \quad (13)$$

Substituting v_1^2 and v_2^2 in equation(2), we obtain the kinetic energy of each link as follows:

$$K_1(\theta,\dot{\theta}) = \frac{1}{2}\,m_1l_1^2\dot{\theta}_1^2 \quad (14)$$

$$K_2(\theta,\dot{\theta}) = \frac{1}{2}\,m_2l_1^2\dot{\theta}_1^2 + \frac{1}{2}\,m_2l_2^2\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\right)$$
$$+ m_2l_1l_2c_2\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right) \quad (15)$$

so that the total kinetic energy of the robot arm is obtained from equations (14) and (15) and presented as

$$K(\theta,\dot{\theta}) = \frac{1}{2}\,(m_1+m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}\,m_2l_2^2\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\right) + m_2l_1l_2c_2\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right) \quad (16)$$

Reference with the datum (zero potential energy) at the axis of rotation, the potential energy of the robot arm is the sum of the potential energies of the link 1 and link 2 which is obtained as follows;

$$U(\theta) = \sum_i^n U_i\,(\theta) \quad (17)$$

$$U(\theta) = -(m_1 + m_2)gl_1c_1 - m_2gl_2c_{12} \quad (18)$$

The Lagrange-Euler formulation defines the behaviour of a dynamic system in terms of works and energy stowed in the system (Urrea & Pascal, 2017). The Lagrangian L is demarcated as;

$$L(\theta,\dot{\theta}) = K - U \quad (19)$$

$$L(\theta,\dot{\theta}) = \frac{1}{2}\,(m_1+m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}\,m_2l_2^2\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\right) + m_2l_1l_2c_2\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right)$$
$$+ (m_1 + m_2)gl_1c_1$$
$$+ m_2gl_2c_{12} \quad (20)$$

From this Lagrangian, the dynamic systems equations of the motion are given by:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = \tau_i \quad (21)$$

where L is the Lagrangian function, K the kinetic energy, U the potential energy, τ_i the generalized coordinates torque exerted on θ_i.

For the coordinateθ_1, the Lagrange's equation are;

$$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2)l_1^2\,\dot{\theta}_1 + m_2 l_2^2(\dot{\theta}_1 + \dot{\theta}_2) + 2m_2 l_1 l_2 c_2 \dot{\theta}_1 + m_2 l_1 l_2 c_2 \dot{\theta}_2 \qquad (22)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} = [(m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 c_2]\ddot{\theta}_1 + [m_2 l_2^2 + m_2 l_1 l_2 c_2]\ddot{\theta}_2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2 \qquad (23)$$

$$\frac{\partial L}{\partial \theta_1} = -(m_1 + m_2)g l_1 s_1 - m_2 g l_2 c_{12} \qquad (24)$$

$$\tau_1 = [(m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 c_2]\ddot{\theta}_1 + [m_2 l_2^2 + m_2 l_1 l_2 c_2]\ddot{\theta}_2 - 2m_2 l_1 l_2 s_2\, \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 s_2\, \dot{\theta}_2^2 + (m_1 + m_2) g l_1 s_1 + m_2 g l_2 s_{12} \qquad (25)$$

Similarly, for the coordinate θ_2, the Lagrange's equations are:

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2(\dot{\theta}_1 + \dot{\theta}_2) + m_2 l_1 l_2 c_2 \dot{\theta}_1 \qquad (26)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2\, \dot{\theta}_1 \dot{\theta}_2 \qquad (27)$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2 s_2 \left(\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2\right) - m_2 g l_2 s_{12} \qquad (28)$$

$$\tau_2 = (m_2 l_2^2 + m_2 l_1 l_2 c_2)\ddot{\theta}_1 + m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 g l_2 s_{12} \qquad (29)$$

The derived dynamic equations can be written in terms of the components of inertial matrix, the centrifugal force and Coriolis force vector and the gravity force respectively and they are presented as;

$$M(1,1) = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 c_2 \qquad (30)$$

$$M(1,2) = m_2 l_2^2 + m_2 l_1 l_2 c_2 \qquad (31)$$

$$M(2,1) = m_2 l_2^2 + m_2 l_1 l_2 c_2 \qquad (32)$$

$$M(2,2) = m_2 l_2^2 \qquad (33)$$

$$C(1,2) = -2m_2 l_1 l_2 s_2\, \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 s_2\, \dot{\theta}_2^2 \qquad (34)$$

$$C(2,1) = m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \qquad (35)$$

$$G(1,1) = (m_1 + m_2) g l_1 s_1 + m_2 g l_2 s_{12} \qquad (36)$$

$$G(2,1) = m_2 g l_2 s_{12} \qquad (37)$$

The dynamic equations for the robot manipulator are usually represented by the coupled non-linear differential equations which was derived from lagrangian method;

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \boldsymbol{G}(\boldsymbol{\theta}) = \boldsymbol{\tau} \qquad (38)$$

3. Control Strategy

Proportional-integral-derivate controller (PID) is implemented for effective control of the robot arm. We need two PID controllers since arm1 and arm2 are dependent on each other; as a matter of fact, there is a strong interaction between the two arms. However, the coupling effect needs to be decoupled so as to gain enough freedom to control each arm freely. The main objective is to make the robot arm to move or stop in the desired position to achieve the stated objective we defined a desired (set point) joint angle θ^d , the objective of robot control is to design the input torque in equation (37) such that the regulation error :

$$\tilde{\theta} = \theta^d - \theta \quad (39)$$

And the PID control law is expressed in terms of error, $\tilde{\theta}$ as:

$$\tau_{PID} = K_p\tilde{\theta} + K_i \int_0^t \tilde{\theta}(t)\, dt + K_d\dot{\tilde{\theta}} \quad (40)$$

θ^d : The desired joint [rad]
θ : The actual joint angle [rad]
θ : Angle error [rad]

The closed-loop system for two-degree-of-freedom robot arm shown Figure 2 which provides insight into the modeling (referred to fig. 3) and the control aspect of the robot arm.

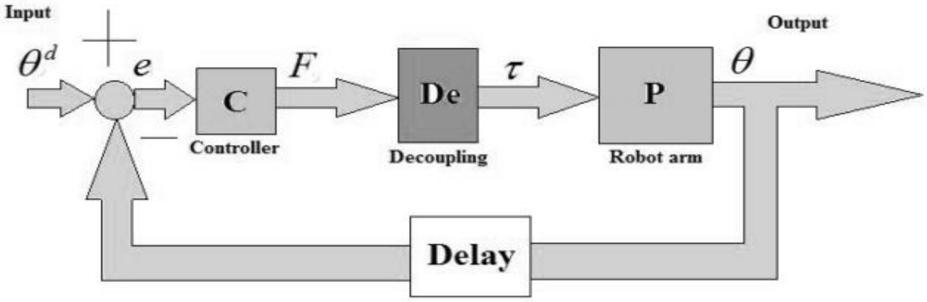


Figure 2: Closed loop system for 2-DOF robot arm control

The closed loop equation of the robot arm is obtained by substituting the control action τ_{PID} in equation (40) into the robot model(38).

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = K_p\tilde{\theta} + \xi + K_d\dot{\tilde{\theta}}\,\tau = \tau_{PID} \quad (41)$$

where $\dot{\xi} = K_i.\tilde{\theta}$ (42)

According to Murray, (1994), the work done by the motion of the end effectors is expressed as

$$W = \int_{t_1}^{t_2} \psi.F\, dt \quad (42)$$

where, W is the work done by the end effector, $t_1\ to\ t_2$ time interval, ψ the linear velocity vector and F the applied force vector of the motion of the end effector.

This work is the same as the work perform by robot arm, so

$$\int_{t_1}^{t_2} (\dot{\theta}\tau)\, dt = W = \int_{t_1}^{t_2} \psi.F\, dt \quad (43)$$

$\dot{\theta}$, the angle velocity vector and τ the applied torque vector and

$$\psi = J.\dot{\theta} \quad (44)$$

Equation 40 is further simplify as:

$$\dot{\theta}^T.\tau = \dot{\theta}^T\, j^T.F \quad (45)$$

It follows that,

$$\tau = j^T.F \quad (46)$$

Let consider the two-degree-of-freedom (2 DOF) robot arm with joint coordinates θ_i where $i = 1$ and $i = 2$ and a Cartesian coordinate x, x is defined as Cartesian corresponding to the joint position vector $\theta = [\theta_1, \theta_2]^T$, and the angle velocity and angle acceleration vectors $\dot{\theta}$ and $\ddot{\theta}$ respectively and let $x = h(\theta)$, where

$$\dot{x} = \frac{\partial h(\theta)}{\partial \theta}.\dot{\theta} \quad (47)$$

Where, $\dot{x}$ is the linear velocity, that is $\psi = \dot{x}$

$$x = h(\theta) = \begin{bmatrix} h_1(\theta) \\ h_2(\theta) \end{bmatrix} \quad (48)$$

$h(\theta)$, the function between the angle position and the Cartesian position and the partial differential of equation (6) and (7) with respect to θ_1,θ_2 result in Jacobian matrix J.

$$J = \frac{\partial h(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial h_1}{\partial \theta_1} & \frac{\partial h_1}{\partial \theta_2} \\ \frac{\partial h_2}{\partial \theta_1} & \frac{\partial h_2}{\partial \theta_2} \end{bmatrix} \quad (49)$$

From the equation(42), F is the output of the PID controller and the Jacobian matrix J is designed as the decoupling part and the corresponding input of the robot arm can be written as:

$$\begin{bmatrix} \tau_{PID_1} \\ \tau_{PID_2} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} F_{PID_1} \\ F_{PID_2} \end{bmatrix} \quad (50)$$

4. Results and Discussion

Simulink model of 2-DOF robot arm is prepared based on the Lagrangian and Lagrange-Euler formulation derived in the equation 1 to 38 and the PID controllers are implemented from the equation 41 and 50. The parameter values for two-degree-of-freedom (2-DOF) robot arm presented in Table 1 are used for the simulation. This model is further split into sub-systems to reduce system complexity and size and latter combined as one model as depicted in Figure 3. In Simulink toolbox PID block is available which is implemented to control the joint angle. The tuning of control parameters is done using PID tuner and the best performance of the controller parameter values are presented in Table 2.

Table 1: Parameters of the 2-DOF Robot Arm

Parameter	Link 1	Link 2	Unit
m	5.00	2.0	$[\mathbf{kg}]$
l	0.34	0.34	$[\mathbf{m}]$
g	9.81	9.81	$[\boldsymbol{ms^{-2}}]$

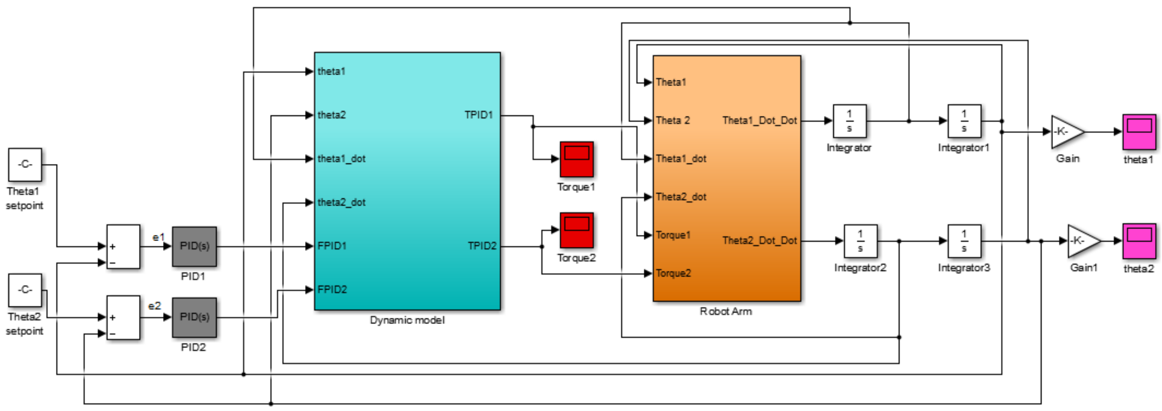


Figure 3: Simulation model for 2-DOF robot arm

Table 2: PID Controller Parameter for 2-DOF Robot Arm

Parameter	Link 1	Parameter	Link 2	Unit
K_{p_1}	30	K_{p_1}	32	[−]
K_{d_1}	12	K_{d_2}	22	[−]
K_{i_1}	20	K_{i_2}	30	[−]

The model is simulated and validate with a different range of joint angles as indicated in Figure 4 and 5. In Figure 4, the initial joint angles for arm 1 and arm 2 are 180° and 90° respectively and the desired joint angle positions we want the arm to reach are 90° and180°. The control strategy ensures that the desired joint angle positions are obtainable by selecting suitable controller gains as indicated in Table 2. It can also be deduced that arm1 and arm 2 followed the desired trajectory as indicated with red and blue line respectively. Furthermore, different angle conditions are selected to ensure that the robot arm performs efficiently, so the initial joint angle of arm 1 is 30° and that of arm 2 is 90° and we expected that the desired joint angle positions should be at 60° and 180° as indicated in the figure (5). It can be deduced that for the robot arm to reach the desired trajectory (angle position), the gains of the PID controller need to be adjusted at every instant and tuned to prevent overshoot and oscillation that associated with changing of parameter values. It can also be observed that the torque applied as shown Figure 6 and 7 slightly overshoot but stabilized quickly. It can be observed that the parameters values influence the controller performance, so adequate online auto tuning of the parameters of the controller will enhance the parameters selection. The model is validated with the work of (Mustafa & Al-Saif, 2014) and the result obtained from the nonlinear model show similar responses with the results presented in this paper. However, the PID decoupling approach adopted in the work of (Mustafa & Al-Saif, 2014) is too rigorous and limited to a specified range of joint angles but the method presented in this research work permit flexibility of joint angles selection and decoupling is dependent on the Jacobian matrix derivation.

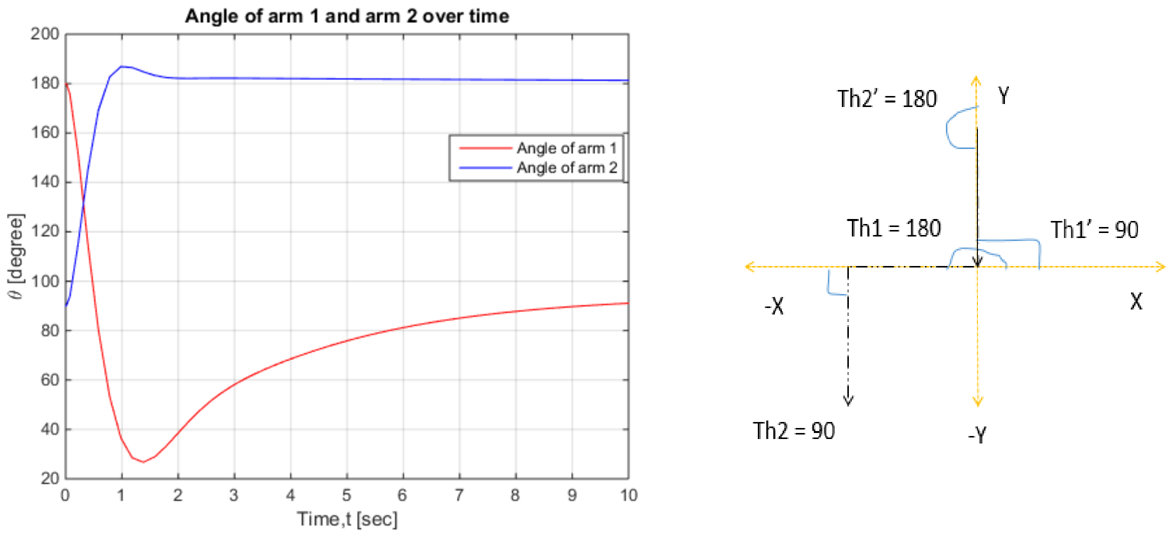


Figure 4: The angle positions of the arm

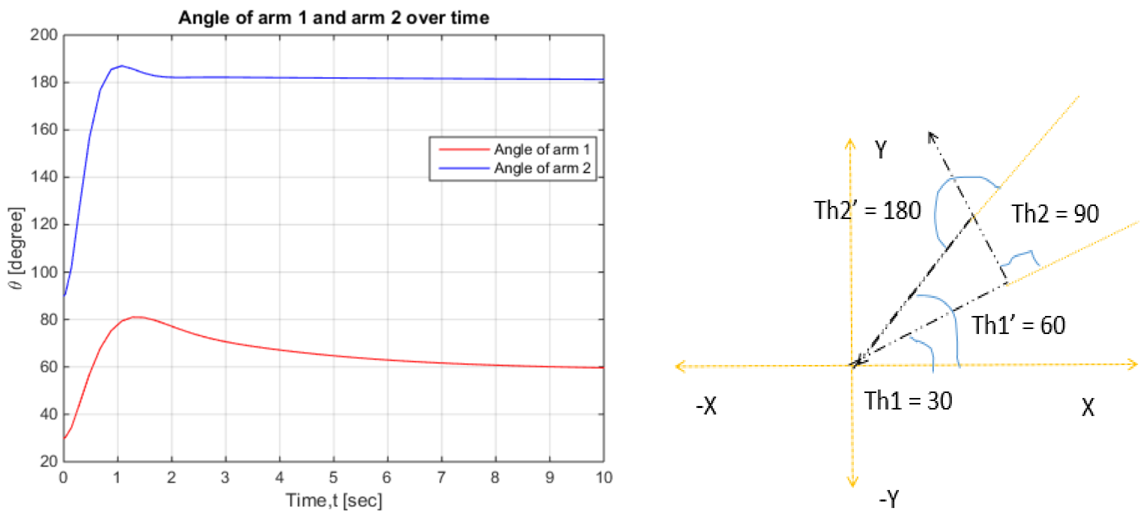
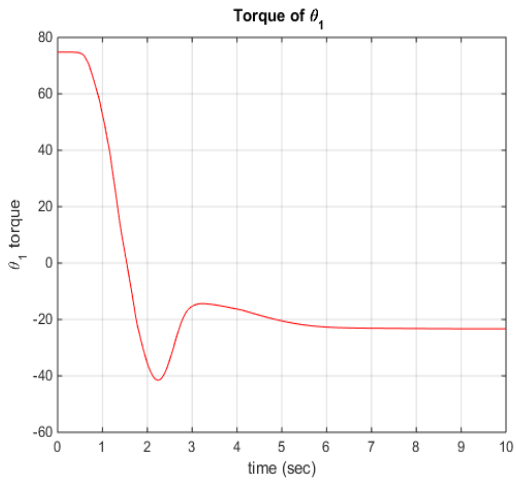


Figure 5: The angle positions of the arm



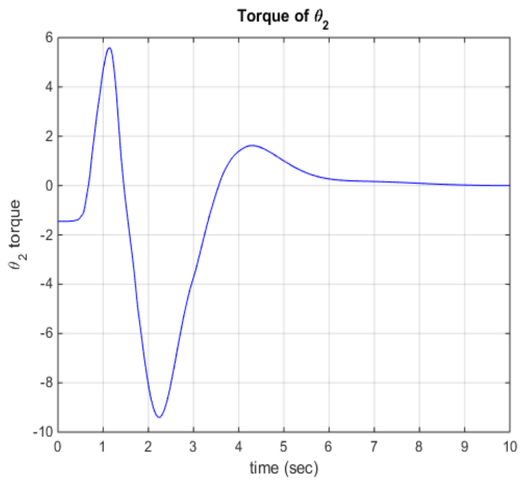


Figure 6: Torque of theta 1

Figure 7: Torque of theta 2

5. Conclusion

In this paper, the mathematical modeling, control and simulation of a 2-DOF robot arm were presented. The distinct feature of this approach was the 2-DOF mathematical model that served as the core element. The approach of using mathematical models, Lagrangian and Euler-Lagrange were to derive a dynamic model that mimicked the actual robot movement in real life scenario and to gain sufficient control over the robot joint positions within the desired trajectory. According to the results analysis, the robot arm was controlled to reach and stay within a desired joint angle position through implementation and simulation of PID controllers using MATLAB/Simulink. Also, the result revealed that changes in initial joint angle positions of the robot arm resulted in different desired joint angle positions and this necessitated that the gains of the PID controllers need to be adjusted and turned at every instant in order to prevent overshoot and oscillation that associated with the change in parameters values. However, an online auto-tuning of the controller parameters can be implemented so as to enhance the parameters selection. As for future work, a more robust control such as H-infinity controller as well PID gain scheduling should be a focus of interest in latest research of robot arm control.

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Nomenclature

$K(\theta,\dot{\theta})$	The Kinetic Energy	[J]
$U(\theta)$	The Potential Energy	[Nm]
$L(\theta,\dot{\theta})$	Lagrangian Formulation	[J]
J	Jacobian Matrix	[m]
θ_i	Joint Angle Position of ith arm	[rad]
$\dot{\theta}_i$	Velocity of ith arm	[rad/s]
$\ddot{\theta}_i$	Acceleration of ith arm	$[rads^{-2}]$
m_i	Mass of each Link	[kg]
l_i	Link lengths	[m]
τ_i	Actuator Torque	[Nm]
g	Acceleration due to gravity	$[ms^{-2}]$
$M(\theta)$	Inertia Matrix	$[N.m.s^{-2}]$
$C(\theta)$	Centrifugal Forces and Coriolis force	
$g(\theta)$	Gravity Force	[Nm]
K_{P_1}	The proportional gain for arm 1	[–]
K_{P_2}	The proportional gain for arm 2	[–]
K_{i_1}	The integral gain for arm 1	[–]
K_{i_2}	The integral gain for arm 2	[–]
K_{d_1}	The derivative gain for arm 1	[–]
K_{d_2}	The derivative gain for arm 2	[–]
τ_{PID}	The Torque of the Controller output	[Nm]
$h(\theta)$	Height of the center of the mass of the ith link	[m]